N90-29335
608777
12 p.

On the Stability of Robotic Systems with Random Communication Rates*

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Abstract

This paper studies control problems of sampled data systems which are subject to random sample rate variations and delays. Due to the rapid growth of the use of computers more and more systems are controlled digitally. Complex systems such as space telerobotic systems require the integration of a number of sub-systems at different hierarchical levels. While many sub-systems may run on a single processor, some sub-systems require their own processor or processors. The sub-systems are integrated into functioning systems through communications. Comunications between processes sharing a single processor are also subject to random delays due to memory management and interrupt latency. Communications between processors involve random delays due to network access and to data collisions. Furthermore, all control processes involve delays due to causal factors in measuring devices and to signal processing.

Traditionally, sampling rates are chosen to meet the worst case communication delay. Such a strategy is wasteful as the processors are then idle a great proportion of the time; sample rates are not as high as possible resulting in poor performance or in the over specification of control processors; there is the possibility of missing data no matter how low the sample rate is picked.

Randomly sampled systems have been studied since later 1950's, however, results on this subject are very limited and they are not applicable to practical systems. This paper studies asymptotical stability with probability one for randomly sampled multi-dimensional linear systems. A sufficient condition for the stability is obtained. This condition is so simple that it can be applied to practical systems. A design procedure is also shown.

1 Introduction

Many complex systems today involve the integration of a number of different subsystems at various hierarchical levels. Examples of hierarchical subsystems are, for example, in the case of spacecraft:

Level 1 – Assignment of systems to tasks;

Level 2 – Assignment of subsystems to task systems, such as the shuttle manipulator, one of more cameras, an astronaut on EVA;

Level 3 – Control of individual subsystems, cameras comprised of pan tilt, zoom, focus, feature tracking, exception warning; or control of machine tools comprised of spindle, table, tool changer, gauge;

Level 4 – Control of elements, control of manipulator joints, end-effector force measurement, machine tool spindle drive, elevator motor drive, submarine plane control.

These systems all comprise many components which may be ranked hierarchically. Many of the components are now computer controlled and are integrated by means of digital busses or networks. The integration

*This material is based upon work supported by the National Science Foundation under Grant No. ECS-11879. Any opinions, findings, conclusions or recommendations expressed in this publication are those of the authors and do not necessarily reflect the views of the National Science Foundation.

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of these components into functioning feedback systems, camera – force-sensor – manipulator roll sensor, pitch sensor – main propulsion – planes, implies in addition to data communications, communication rates.

In the case of communication networks, the data rates of point to point communication busses are well known. The rates at which a computer can respond to communication data interrupts requests add a variance to the data rates. In the case of shared networks, such as Ethernet, data collisions add considerable variance to the data rate frequently exceeding the data rate itself. However, these shared networks are very attractive from both the reliability and flexibility standpoints.

Because of their flexibility in programming and speed in computing, digital computers are now regularly employed as integral components of dynamic feedback control systems. They are easily programmed to realize desired compensators. Due to the discrete nature of digital computers, variables in dynamic systems are sampled and quantized before sending to the computers. The well established discrete time system theory (e.g., [8]) provides methods to analyze the behavior of sampled data systems, based on the assumption that the sampling rates are fixed and the same, and the sampling operations on different channels of the systems are synchronized. If the sampling rates are fixed but different on different channels, known as multi-rate sampling, the system analyses are simple if the sampling rates have integral ratios [6, 10].

Due to random delays in measurement devices, signal processing, interrupt latency, priority scheduling, conditional branching, network communications, etc., sampling rates vary randomly in many systems, and the system performance could be expected to be improved if a theory supporting random sampling rates was used. Systems with random sampling processes are called randomly sampled systems. The behavior of a randomly sampled system is, presumably, related to the statistical properties of the random sampling processes as well as system parameters. Randomly sampled systems have been studied by Kalman [11], Leneman [16], Kushner and Tobias [15], Agniel and Jury [2], and others. One of the major motivations for studying randomly sampled systems in late 1950's and early 1960's was the introduction of digital computers in control systems. However as the speed of computers improved dramatically, time delays caused by computers became practically negligible in simple single processor controlled systems compared to other delays, and research on randomly sampled systems came to an end. Nowadays, development of computer controlled systems has reached beyond the stage of single processor control. Many subsystems are integrated into large systems. Furthermore, many complex dynamic systems impose demanding computation requirement. For example, computation time becomes a bottleneck in the implementation of dynamic control algorithms of multi-joint robot manipulators. Delay caused by computation and communication is no longer a negligible factor.

Early researchers in the area of randomly sampled systems primarily considered stability conditions of the systems. Their work is briefly summarized below. Kalman carried out a comprehensive study of sampling systems [11]. He classified sampling into six categories: conventional sampling, nonsynchronous sampling, multiple-order sampling, multi-rate sampling, noninstantaneous sampling, and random sampling. For randomly sampled systems, Kalman showed that if the second moment of the output of an autonomous system is stable, the second moment of the output remains bounded when a bounded input is applied to the system. Based on his state space method [13], Kalman [12] also discussed the regulator problem and stability of a linear system described by independent random functions. This class of systems include randomly sampled systems. Thus the stability conditions obtained for this class of systems are applicable for randomly sampled systems. Kushner and Tobias [15] studied an autonomous linear system with linear and nonlinear feedback. Using a stochastic Lyapunov function, criteria for stability with probability one and s-th moment stability ($s > 0$) were obtained for scalar linear systems, and criteria for stability with probability one and second moment stability were obtained for multi-dimensional linear systems. Agniel and Jury [2] investigated asymptotic stability with probability one of a linear system with a saturating type nonlinear component. A computational procedure was provided to determine the largest stability sector of the nonlinearity for asymptotic stability with probability one. Using a stochastic Lyapunov function, Agniel and Jury in another paper [1] gave a condition for the asymptotic stability with probability one and the second moment asymptotic stability for single-input single-output multi-dimensional linear systems. They also showed that if an autonomous system exhibits asymptotic stability with probability one, the system is almost surely bounded input-bounded output. Leneman [16] studied a single-input single-output first order linear system with feedback. He derived the second moment of the output for the cases with and without input. The input is a stationary stochastic process independent of the sampling process. Consequently, a condition for the second moment stability was given. Assuming the independence of the sampling times and the signals, Dannenberg and Melsa [7] took the expectation of a linear system equation, obtaining a system equation of expectation of the states and outputs. The first moment stability analysis is similar to that of deterministic sampled-data systems. An example of a spacecraft control problem was given, in which it is assumed that there is a probability of missing messages. The problem of random sampling of a random signal was studied by Bergen [4] and Leneman [17]. Their focus was on deriving expressions of the spectral density of a random signal after a random sampling.

This paper studies the stability of randomly sampled systems in relation to the random sampling processes. Though Kalman [11] and Kushner [15] have obtained necessary and sufficient conditions for the stability in the second moment, it is not so easy to apply these conditions to practical systems. This paper studies asymptotic stability with probability one and gives a necessary and sufficient condition for one-dimensional systems and a sufficient condition for multi-dimensional systems. These conditions are easy to verify for given sampling distributions and are thus applicable to practical systems.

In the next section, the asymptotical stability with probability one is defined. A sufficient condition is given for multi-dimensional linear time-invariant randomly sampled systems which is also necessary for one-dimensional systems. A design procedure todetermine feedback gains is obtained in Section 3. If we use a nonlinear compensator such as a computed torque controller for a robotic control system, then we would have a set of simple two-dimensional linear systems. In Section 4, the stability of such two-dimensional systems is considered and the design prcedure is shown for a Bernoulli distribution, a uniform distribution and a mixed uniform distribution.

2 Stability

Consider following linear time-invariant control system.

$$\dot{x}(t) = Ax(t) + Bu(t), \tag{1}$$

where x is an n-dimensional state vector, u an r-dimensional control vector, and A and B are $n \times n$ and $n \times r$ matrices, respectively. For this system, we apply a constant state feedback input

$$u(t) = Kx(t_k), \tag{2}$$

from $t = t_k$ to $t = t_{k+1} (= t_k + \Delta_k)$, where K is an $r \times n$ matrix. Then $x(t_{k+1})$ is given as follows.

$$x(t_{k+1}) = (\Phi(\Delta_k) + \Psi(\Delta_k)K)x(t_k), \tag{3}$$

where

$$\Phi(\Delta_k) = \exp(A\Delta_k), \text{ and } \Psi(\Delta_k) = \int_0^{\Delta_k} \exp(A\tau)d\tau B.$$

Sampling interval Δ_k is assumed to be subject to some probability distribution function $F(\Delta)$ or distribution density function $f(\Delta)$ and Δ_i and $\Delta_j (i \neq j)$ are statistically independent of each other. For simplicity, we write Eq. (3) as follows

$$x_{k+1} = \Gamma(\Delta_k)x_k. \tag{4}$$

In this paper, we use the following matrix norm which is compatible with usual Euclid norm for vectors:

$$||\Gamma|| = \{\sigma(\Gamma^*\Gamma)\}^{1/2}, \tag{5}$$

where Γ^* is the conjugate transformed matrix and $\sigma(\Gamma)$ denotes the maximum eigenvalues of the matrix Γ. Note, however, that while the stability of the system (1) or (3) is invariant under a similarity transformation of the state variables, the matrix norm depends on the transformation, namely in general

$$||\Gamma|| \neq ||T^{-1}\Gamma T||.$$

The stability of randomly sampled control system Eq. (1) is defined as follows.

Definition 1 (Stability) *The randomly sampled control system Eq. (1) is asymptotically stable with probability one if*

$$Prob[\lim_{k\to\infty} ||x_k|| = 0] = 1$$

for any initial state x_0, where $||x||$ is the Euclid norm of vector x.

Now we define the following notation:

$$E[\omega] \quad : \text{Expectation of random variable } \omega,$$

$$V[\omega] \quad : \text{Variance of random variable } \omega,$$

and assume that

$$E[\{\log(||\Gamma(\Delta)||)\}^2] < \infty. \tag{6}$$

Then a sufficient condition of the asymptotical stability is given in the next proposition.

Proposition 1 (Sufficient Condition) *Randomly sampled control system (1) is asymptotically stable with probability one if*

$$E = E[\log(||T^{-1}\Gamma(\Delta)T||)] < 0, \tag{7}$$

We also have

$$Prob[||T^{-1}x_k|| < ||T^{-1}x_0|| \exp\{k(E+\epsilon)\}] > 1 - \frac{V}{k\epsilon^2}, \tag{8}$$

for any $\epsilon > 0$, where $V = V[\log(||T^{-1}\Gamma(\Delta)T||)]$.

< proof > Assuming $x_0 \neq 0$ without loss of generality, from Eq. (4) we have

$$\log(||T^{-1}x_k||/||T^{-1}x_0||) \leq \sum_{i=0}^{k-1} \log(||T^{-1}\Gamma(\Delta_i)T||).$$

Then the proposition is easily proved by the statistical independence of Δ_i's and Thebyshev's inequality.

< end of proof >

We note that for one-dimensional systems the condition stated in the above proposition is *necessary and sufficient* for the aymsptotic stability with probability one [14]. If the sampling interval is constant, the condition in Prop. 1 is also necessary for the asymptotic stability of multi-dimensional systems.

Now we define

$$\gamma(\Delta) = \log(||T^{-1}\Gamma(\Delta)T||), \qquad \text{and} \qquad g(\Delta) = \int_0^\Delta \gamma(\tau)d\tau, \tag{9}$$

then we have the following proposition.

Proposition 2

i. *If the sampling rate Δ is subject to a Bernoulli distribution where $\Delta = \alpha$ with probability p and $\Delta = \beta$ with probability $q = 1 - p$, then the system is asymptotically stable with probability one, if*

$$p\gamma(\alpha) + q\gamma(\beta) < 0.$$

ii. *If the sampling rate Δ is subject to a uniform distribution $\mathcal{U}[\alpha, \beta]$, then the system is asymptotically stable with probability one, if*

$$g(\alpha) < g(\beta).$$

iii. *If the sampling rate Δ is subject to $\mathcal{U}[\alpha, \beta]$ with probability ε and to $\mathcal{U}[\mu, \nu]$ with probability $1 - \varepsilon$, then the system is asymptotically stable with probability one, if*

$$\varepsilon\frac{g(\beta) - g(\alpha)}{\beta - \alpha} + (1 - \varepsilon)\frac{g(\nu) - g(\mu)}{\nu - \mu} < 0.$$

The proof is straightforward, so we omit it here.

3 Design Procedure

Next we discuss a design procedure of a feedback gain K and a matrix T in the following. Now, assume that system

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{10}$$

is controllable, then it is well known that the discretized system

$$x_{k+1} = \Phi(\Delta_k)x_k + \Psi(\Delta_k)u_k \tag{11}$$

is also controllable for almost all sampling interval Δ_k [5]. Then we can assign poles $\{\lambda_i, \, i = 1, 2, \ldots, n\}$ to system (11) if poles $\{\lambda_i\}$ are symmetric with respect to the real axis. Here, we apply Hikita's pole assignment algorithm[9] to the randomly sampled control systems.

[Algorithm]

step (i) *For given $\{\lambda_i\}$, find r-dimensional vectors ξ_i, $i = 1, 2, \ldots, n$, which makes matrix $T(\hat{\Delta}) = [v_1 : \cdots : v_n]$ non-singular. Vector v_i's are given as follows where $\Phi = \Phi(\hat{\Delta})$ and $\Psi = \Psi(\hat{\Delta})$.*

- *if λ_i is a real number, then*

$$v_i = (\Phi - \lambda_i I_n)^{-1}\Psi\xi_i. \tag{12}$$

- *if λ_i and λ_{i+1} are conjugate complex numbers $\alpha_i \pm j\beta_i$, then*

$$v_i = V_{1i}\xi_i - V_{2i}\xi_{i+1}, \text{ and } v_{i+1} = V_{1i}\xi_i + V_{2i}\xi_{i+1}, \tag{13}$$

where

$$V_{1i} = \{(\Phi - \alpha_i I_n)^2 + \beta_i^2 I_n\}^{-1}(\Phi - \alpha_i I_n)\Psi, \text{ and } V_{2i} = \{(\Phi - \alpha_i I_n)^2 + \beta_i^2 I_n\}^{-1}\beta_i\Psi. \tag{14}$$

step (ii) *Feedback gain K is given as follows.*

$$K(\hat{\Delta}) = -[\xi_1 : \cdots : \xi_n]T(\hat{\Delta})^{-1}. \tag{15}$$

step (iv) *Check the stability using Proposition 1 or 2. If not stable, return step (i) and try another $\{\lambda_i\}$ and/or $\hat{\Delta}$.*

It is easy to show that for this $T(\hat{\Delta})$ and $K(\hat{\Delta})$, we have

$$||T^{-1}(\hat{\Delta})\Gamma(\hat{\Delta})T(\hat{\Delta})|| = \max_i\{|\lambda_i|\}. \tag{16}$$

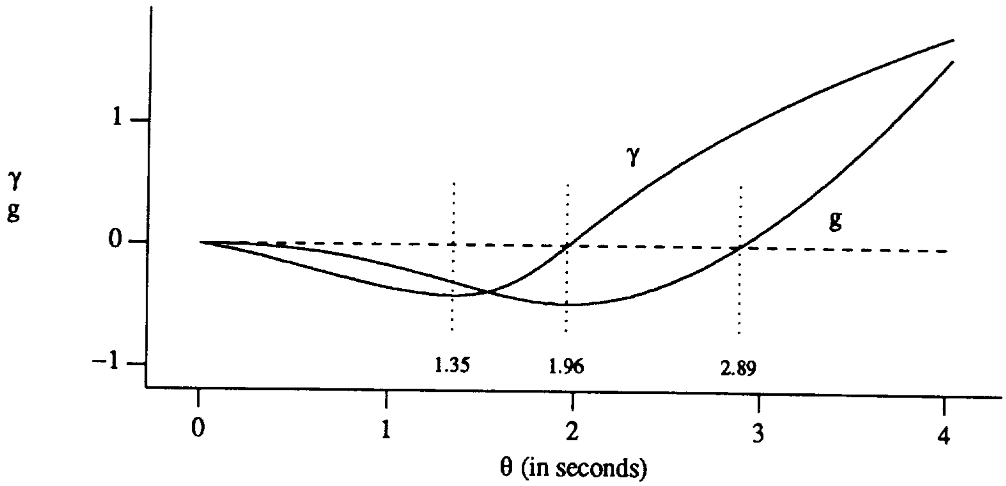
Hence we can use matrices $T(\hat{\Delta})$ and $K(\hat{\Delta})$ to calculate $\gamma(\Delta)$ and $g(\Delta)$. In the next section, we use notations $\gamma(\Delta, \hat{\Delta})$ and $g(\Delta, \hat{\Delta})$ for $\gamma(\Delta)$ and $g(\Delta)$, respectively, to show the dependence of the functions on $\hat{\Delta}$ clearly.

4 Two Dimensional Systems

In this section, we consider control of robot manipulators. We view a robot manipulator as a component of a large system, such as a space station. The robot controller communicates with the other components of the system to achieve cooperative actions. Communication between components is considered to have a longer delay than that within a component. We assume that robot controller has an inner feedback loop which compensates the nonlinearity of manipulator dynamics and operates independently of the other part of the system. The robot dynamic system together with the inner feedback loop becomes a linear system. It is feasible to treat the robot manipulator subsystem as a linear system when integrating and communicating with the other components. For example, if we use the nonlinear feedback controller developed in [3], we have n (=DOF of manipulator) decoupled two-dimensional linear systems

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \tag{17}$$

Figure 1: function $\gamma(\theta, 1)$ and $g(\theta, 1)$



where $x(t) = (e_i(t), \dot{e}_i(t))$ is the error vector for the i-th component of outputs and $u(t)$ is the corresponding input for this component of outputs. If the task is specified in joint space (the joint space control), the i-th component of output is simply the displacement of the i-th joint and the error vector is composed of the joint position error and joint velocity error.

We now study the asymptotical stability of this system under the random sampling rate. The corresponding discrete time system is easily obtained for a sampling interval Δ as follows.

$$x_{k+1} = \begin{bmatrix} 1 & \Delta_k \\ 0 & 1 \end{bmatrix} x_k + \begin{bmatrix} \Delta_k^2/2 \\ \Delta_k \end{bmatrix} u_k. \tag{18}$$

We apply the algorithm given above to this system directly. Then we have the following proposition.

Proposition 3 (PD Controller) *Assume that* $\{\lambda_i\} = \{\lambda_1, \lambda_2\}$ *where* $\lambda_1 \neq \lambda_2$, *then we have*

$$K(\hat{\Delta}) = ((\lambda_1 + \lambda_2 - \lambda_1\lambda_2 - 1)/\hat{\Delta}^2, (\lambda_1 + \lambda_2 + \lambda_1\lambda_2 - 2)/(2\hat{\Delta})),$$

and

$$\gamma(\Delta, \hat{\Delta}) = \gamma(\theta, 1),$$

where $\theta = \Delta/\hat{\Delta}$.

The proof is obtained by direct calculation. This proposition implies that the function $\gamma(\Delta, \hat{\Delta})$ is the same as the function $\gamma(\theta, 1)$ if we use $K(\hat{\Delta}) = (k_p/\hat{\Delta}^2, k_v/\hat{\Delta})$ instead of $K(1) = (k_p, k_v)$. Therefore we have $g(\Delta, \hat{\Delta}) = \hat{\Delta} g(\theta, 1)$ for the same $K(\hat{\Delta})$. This fact is very useful to design the feedback gain. This will be shown by examples.

Fig. 1 shows $\gamma(\theta, 1)$ and $g(\theta, 1)$ for $\lambda_1 = 0.4$ and $\lambda_2 = 0.7$, where we have

$$K(1) = -(0.18, 0.81), \text{ and } T(1) = \begin{bmatrix} -0.759 & -0.934 \\ 0.651 & 0.333 \end{bmatrix},$$

and ξ_i was used to make the norm of column vectors of T matrix be equal to one.

Example 1 (Bernoulli Distribution) *Let's assume that the sampling interval is subject to Bernoulli distribution, i.e.* $\Delta = \alpha$ *with probability* p *and* $\Delta = \beta$ *with probalility* q, *where* $\alpha < \beta$, $0 \leq p \leq 1$, *and* $q = 1 - p$. *The sufficient stability condition is given as follows.*

$$p\gamma(\alpha/\hat{\Delta}, 1) + q\gamma(\beta/\hat{\Delta}, 1) < 0. \tag{19}$$

Note that if $\hat{\Delta} \geq \beta/1.96 (= \hat{\Delta}^*)$ *then the system is asymptotically stable for any* α *because* $\gamma(\theta, 1) < 0$ *for any* $\theta \leq 1.96$. *But we are generally interested in the smallest* $\hat{\Delta}$ *because it gives us the fastest response.*

Fig. 1 shows that the function $\gamma(\theta,1)$ reaches the minimum value -0.417 at $\theta = 1.35$. Let θ^* be the point which satisfies the following equation.

$$\gamma(\theta^*,1) = \frac{p}{q} \times 0.417.$$

Then it is clear that $\hat{\Delta}$ must be greater than $\hat{\Delta}_{min}(= \beta/\theta^*)$ for Eq. (19).

A suitable value of $\hat{\Delta}$ can be found from the range $\hat{\Delta}_{min} < \hat{\Delta} < \hat{\Delta}^*$ by a trial-and-error method using Fig. 1 or Table 1 which gives pairs of $\{\theta_1,\theta_2\}$ such that $\gamma(\theta_1,1) = \gamma(\theta_2,1)$.

(i) *Calculate $a = -(q/p)\gamma(\beta/\hat{\Delta},1)$.*

(ii) *Find $\{\theta_1,\theta_2\}$ such that $\gamma(\theta_1,1) = \gamma(\theta_2,1) \leq a$ using Fig. 1 or Table 1.*

(iii) *Check $\theta_1 < \alpha/\hat{\Delta} < \theta_2$. If so, calculate $K(\hat{\Delta})$. If not so, go back to step (i) with another $\hat{\Delta}$.*

For example, if $\alpha = 10$ msec, $\beta = 30$ msec, and $p = 0.75$, then θ^* is about 3.64 and $\hat{\Delta}_{min} = 8.24$ msec, while $\hat{\Delta}^* = 15.3$ msec. If we select $\hat{\Delta} = 11$ msec then $\frac{q}{p}\gamma(\beta/\hat{\Delta},1) = -0.278$ and $\alpha/\hat{\Delta} = 0.91$. Therefore we can try the 6-th row of Table. 1, and we have $\theta_1 = 0.84 < 0.91 < \theta_2 = 1.68$. Hence the system is asymptotically stable for $K = -(1488, 73.64)$.

Example 2 (Uniform Distribution) *Now assume that Δ is subject to a uniform distribution $\mathcal{U}[\alpha,\beta]$. The sufficient condition of the asymptotical stability with probability one is given as follows:*

$$g(\alpha/\hat{\Delta},1) > g(\beta/\hat{\Delta},1).$$

The function $g(\theta,1)$ has its minimum value at $\theta = 1.96$. Now we define $\hat{\Delta}^* = \beta/19.6$ and $\hat{\Delta}_{min} = \beta/2.89$. If $\hat{\Delta} \geq \hat{\Delta}^*$, then the above sufficient condition is satisfied for any α. Therefore the system is asymptotically stable if $\hat{\Delta} \geq \hat{\Delta}^*$. On the other hand, if $\hat{\Delta} \leq \hat{\Delta}_{min}$, then the above condition is not satisfied for any α.

Table 1 also gives pairs of $\{\theta_3,\theta_4\}$ and the ratio θ_3/θ_4 such that $g(\theta_3,1) = g(\theta_4,1)$. If there is a pair $\{\theta_3,\theta_4\}$ such that $\alpha/\beta > \theta_3/\theta_4$, then the system is asymptotically stable for the $K(\hat{\Delta})$ where $\hat{\Delta} = \alpha/\theta_3$. Therefor we can determine $\hat{\Delta}$ easily using this table as follows:

(i) *Calculate $a = \alpha/\beta$.*

(ii) *Find a pair $\{\theta_3,\theta_4\}$ in the Table 1 such that $a > \theta_3/\theta_4$.*

(ii) *Calculate $\hat{\Delta} = \alpha/\theta_3$ and $K(\hat{\Delta})$.*

Now assume that $\alpha = 10$ msec and $\beta = 30$ msec, then we have $\hat{\Delta}^* = 15.3$ msec, $\hat{\Delta}_{min} = 10.38$ msec, and $\alpha/\beta = 1/3 > 0.273$ in the Table 1. Therefore we can use $\alpha/\hat{\Delta} = 0.75$ and $\hat{\Delta} = 13.33$msec. Hence the system is asymptotically stable with $K = -(1065, 62.31)$ if $\beta < 36.7$ msec. Table 2 shows the IAE (Integration of Absolute value of the Error) for fifty random streams with the initial condition $x(0) = (1.0, 0)^T$. The table shows that when $\beta \geq 40$ msec, the STD (STanderd Deviation) and the maximum values of IAE for the velocity error $\dot{e}_i(t)$ become very large compared to the cases where $\beta \leq 35$ msec. This means that the system is still stable but there is a large vibration in the response for $\hat{\Delta} \geq 40$ msec. It is interesting since $\hat{\Delta}$ selected above assures the asymptotically stability for $\beta < 36.7$ msec.

Example 3 (Mixed Uniform Distribution) *Next we assume that Δ is subject to a uniform distribution $\mathcal{U}[\alpha,\beta]$ with probability ε and to $\mathcal{U}[\mu,\nu]$ with probability $1-\varepsilon$. The sufficient condition is given as follows:*

$$E = \varepsilon\frac{g(\beta/\hat{\Delta},1) - g(\alpha/\hat{\Delta},1)}{\beta/\hat{\Delta} - \alpha/\hat{\Delta}} + (1-\varepsilon)\frac{g(\nu/\hat{\Delta},1) - g(\mu/\hat{\Delta},1)}{\nu/\hat{\Delta} - \mu/\hat{\Delta}} < 0.$$

Though the selection of $\hat{\Delta}$ becomes a little difficult, we can use the following procedure to estimate an appropriate $\hat{\Delta}$:

(i) *Define $\bar{\alpha} = (\alpha+\beta)/2.0$, $\bar{\beta} = (\mu+\nu)/2.0$, $p = \varepsilon$, and $q = 1-p$.*

(ii) *Determine $\hat{\Delta}$ using the procedure in Exam. 1 for $\alpha = \bar{\alpha}$ and $\beta = \bar{\beta}$.*

(iii) *Check the condition. If satisfied, calculate $K(\hat{\Delta})$. If not, try another value for $\hat{\Delta}$.*

Figure 2: Simulations for Bernoulli Distribution, Uniform Distribution and Mixed Uniform Distribution

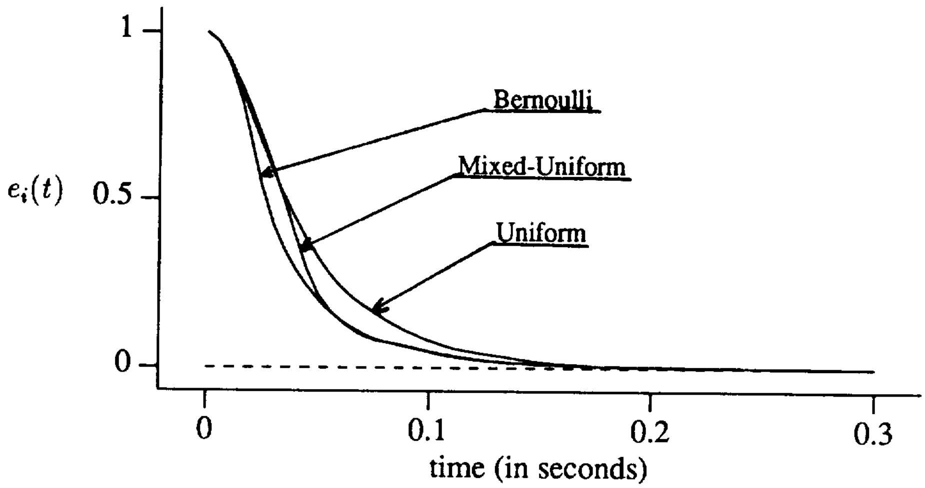


Table 1: θ_1, θ_2 ,θ_3, and θ_4

$\gamma(\theta_1,1) = \gamma(\theta_2,1)$			$g(\theta_3,1) = g(\theta_4,1)$			
$\gamma(\theta,1)$	θ_1	$\theta 2$	$g(\theta,1)$	θ_3	θ_4	θ_3/θ_4
0.00	0.00	1.96	0.000	0.00	2.88	0.000
-0.05	0.18	1.92	-0.009	0.25	2.87	0.087
-0.10	0.33	1.89	-0.039	0.50	2.84	0.176
-0.15	0.46	1.83	-0.094	0.75	2.78	0.270
-0.20	0.58	1.79	-0.173	1.00	2.69	0.372
-0.25	0.71	1.73	-0.270	1.25	2.56	0.488
-0.30	0.84	1.68	-0.373	1.50	2.39	0.628
-0.35	0.98	1.60	-0.456	1.75	2.17	0.806
-0.40	1.18	1.48	-0.467	1.80	2.12	0.849

Now assume that Δ is subject to $\mathcal{U}[5\,msec, 15\,msec]$ with probability $\varepsilon = 0.75$ and to $\mathcal{U}[20\,msec, 40\,msec]$ with probability 0.25. Then we have $\bar{\alpha} = 10msec$, $\bar{\beta} = 30msec$, $p = 0.75$, and $q = 0.25$. If we use $\hat{\Delta} = 11msec$ from the result of Exam. 1, then we have $E = -0.04 < 0$. Therefore the system is asymptotically stable for the same $K = -(1488, 73.64)$.

Fig. 2 shows the simulations of $x(t)$ for three cases discussed above where $x(0) = (1.0, 0)^T$.

It is easily shown that even if we use a PID controller

$$z_{k+1} = z_k + [1:0]x_k, \text{ and } u_k = K_1 z_k + K_2 x_k, \tag{20}$$

or a PD controller with one step delay

$$u_k = K(\Phi(\hat{\Delta})x_{k-1} + \Psi(\hat{\Delta})u_{k-1}), \tag{21}$$

instead of the PD controller given in Prop. 3, we have the similar proposition . Therefore we can determine $\hat{\Delta}$ easily.

5 Conclusions

In this paper, the stability of randomly sampled linear control systems was discussed and the following results were obtained.

1. A sufficient condition for the asymptotical stability in a norm with probability one was obtained for multi-dimensional systems.

2. For a simple two-dimensional system with PD controllers, a design procedure was shown which was easily applicable to systems with PID controllers or PD controllers with one step delay.

The results given in this paper are also easily applicable to the robotic control systems where computed torque controllers or PD controllers with a feedforward term are used at the random sampling rate. The results will be shown in the near future [14].

Table 2: IAE for $\mathcal{U}[10\,msec, \beta\,msec]$

	MEAN		STD		MAX	
β	$e_i(t)$	$\dot{e}_i(t)$	$e_i(t)$	$\dot{e}_i(t)$	$e_i(t)$	$\dot{e}_i(t)$
25	0.0531	0.9999	0.0022	0.0011	0.0572	1.0020
30	0.0511	1.0039	0.0043	0.0200	0.0561	1.1310
35	0.0498	1.0198	0.0060	0.0515	0.0589	1.2370
40	0.0509	1.2418	0.0077	0.4305	0.0717	2.7051
45	0.0709	2.6492	0.0638	4.4592	0.4354	30.276

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